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CORRELATION GRAPHS FOR SUPERSONIC FLOW AROUND RIGHT CIRCULAR CONES AT ZERO YAW IN AIR AS A PERFECT GAS

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SUMMARY

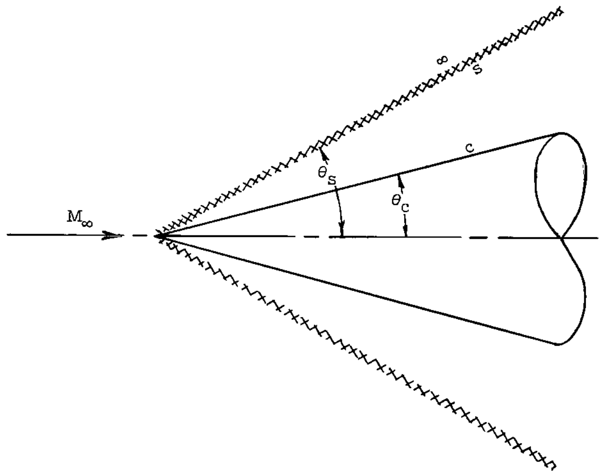
Concise accurate graphs of cone flow properties at zero yaw in air as a perfect gas are presented in correlation form. From these graphs the pressure, density, temperature, shock angle, and Mach number may be obtained both at the cone surface and immediately behind the shock, for free-stream Mach numbers from that for shock detachment to that approaching infinity and for cone semi-apex angles up to 50°. In addition, the initial slope of the normal-force curve is given for the same range of conditions.

INTRODUCTION

Knowledge of the flow about flat plates and cones in supersonic flow is basic to understanding the flow about more complex shapes as well as being important for its own sake. In reference 1 generalized correlation parameters were found to be useful tools for constructing concise accurate graphs describing the two-dimensional oblique shock. For the conical shock no really satisfactory graphs for air exist. For example, the graphs of reference 2 extend only to a Mach number of 4 and those of reference 3 cannot be read accurately at low cone angles and high Mach numbers. There are tabular results for cones available in references 4 and 5. In the present paper these tabular results have been plotted in correlation form to provide accurate readily used graphs for the inviscid flow about cones at zero yaw in air. These charts allow both accurate interpolation and extrapolation. The hypersonic Mach numbers have been favored but wherever possible the entire range of tabulated Mach numbers has been presented. In addition, for usefulness in aerodynamic work these charts have been supplemented by the initial slope of the normal-force-coefficient curve from references 6 and 7.

SYMBOLS

The following sketch is presented to clarify the symbols defined in this section.



C_L lift-force coefficient, $\dfrac{L}{\left(\frac{1}{2}\rho_\infty U_\infty^2 \pi r_b^2\right)}$

C_N normal-force coefficient, $\dfrac{N}{\left(\frac{1}{2}\rho_\infty U_\infty^2 \pi r_b^2\right)}$

C_p surface-pressure coefficient, $\dfrac{(p_c - p_\infty)}{\left(\frac{1}{2}\rho_\infty U_\infty^2\right)}$

$K_c \equiv M_\infty \sin \theta_c$

$K_s \equiv M_\infty \sin \theta_s$

L lift force

M Mach number

N normal force

p static pressure

r_b base radius of cone

T	static temperature
U	velocity
α	angle of attack
γ	ratio of specific heat at constant pressure to that at constant volume
θ_c	semiapex angle of cone
θ_s	semiapex angle of shock wave
ρ	density

Subscripts:

c	cone surface conditions
s	immediately behind shock wave
∞	free-stream conditions
o	zero angle of attack

DISCUSSION OF RESULTS

The tabular results presented in references 4 to 7 have been utilized to construct the charts of inviscid cone flow properties presented as figures 1 to 5. Approximate solutions have been used to obtain the proper form for correlation - notably, that of reference 8 plus supplemental indications from Newtonian impact theory as given in reference 9. The tabulations in references 4 and 6 are for $\gamma = 1.405$ and those in references 5 and 7 are for $\gamma = 7/5$. In most cases this difference in specific-heat ratio was not important; however, in one plot of shock-angle parameter, presented as figure 2(c), it had a significant effect and the results in this figure only apply for $\gamma = 7/5$.

Figure 1 presents the similarity cone-surface-pressure coefficients as a function of the hypersonic similarity parameter. Good correlation is evident beginning at the higher supersonic Mach numbers (generally M_∞ in the range 3 to 4). In this form approximate theories indicate that the pressure parameter is insensitive to the value of γ for which the calculations are made. Comparison with exact calculations for cones in helium (from ref. 10) indicates this to be true. Figure 1(b) allows extremely accurate values to be obtained in the hypersonic range and comparison with the hypersonic cone theory from reference 8.

Correlation of shock angle is presented in figure 2. Figure 2(a) is essentially the difference between the shock angle and the Mach angle in correlation form. Interestingly, it is only for values of K_c greater than about 0.25 that the shock angle is greater than 1 percent larger than the Mach angle. For large values of K_c (the hypersonic case), the shock lies close to the cone so that the angle between the two is not accurately found from this chart. The angle between the cone and the shock (in correlation form) is thus given in figure 2(b) from which more accurate shock angles can be obtained at the larger values of K_c, though less accurate shock angles are obtained at small values of K_c. Even figure 2(b) is inadequate in the respect that accurate extrapolation cannot be done. Thus, figure 2(c) was prepared which is the shock-angle analog of the pressure results in figure 1(b) and from which accurate results can be obtained for the hypersonic range. Again, the hypersonic cone theory from reference 8 is included for comparison. This graph unlike the others in this paper applies only for $\gamma = 7/5$ since the effect of a small change in γ is readily detectable to the scale that is used.

The density, temperature, and Mach number at the cone surface are presented in figure 3. The temperature ratios given in figure 3(b) may be extrapolated with an accuracy of ± 0.2 percent for $K_c \geqq 20$ and $0^\circ \leqq \theta_c \leqq 25^\circ$ by the equation

$$\frac{T_c}{T_\infty} - 1 = 0.2360 K_c^{\,2}$$

The ratios of the pressure, density, temperature, and Mach number at the cone surface to those immediately behind the shock are shown in figure 4. Figure 4(e) allows accurate values of this Mach number ratio to be obtained in the hypersonic range.

The asymptotic value of p_c/p_s as $M_\infty \to \infty$ can be obtained from hypersonic cone theory (ref. 8) as

$$\frac{p_c}{p_s} \to \frac{\gamma + 7}{8} \; ; \text{ thus, } \quad \frac{\rho_c}{\rho_s} \to \left(\frac{\gamma + 7}{8}\right)^{1/\gamma} \quad \text{and} \quad \frac{T_c}{T_s} \to \left(\frac{\gamma + 7}{8}\right)^{(\gamma - 1)/\gamma}$$

so that the asmyptotic relation for the ratio of gas surface density to free-stream density (fig. 3(a)) is

$$\frac{\rho_c}{\rho_\infty} \to \left(\frac{\gamma + 1}{\gamma - 1}\right)\left(\frac{\gamma + 7}{8}\right)^{1/\gamma}$$

The initial slope of the normal-force-coefficient curve in correlation form is given in figure 5. Figure 5(b) allows accurate values to be obtained in the hypersonic range. The initial slope of the lift-coefficient curve (inviscid) can be obtained as follows

$$\left(\frac{dC_L}{d\alpha}\right)_o = \left(\frac{dC_N}{d\alpha}\right)_o - \frac{C_p}{57.296}$$

where α is in degrees.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., March 24, 1964.

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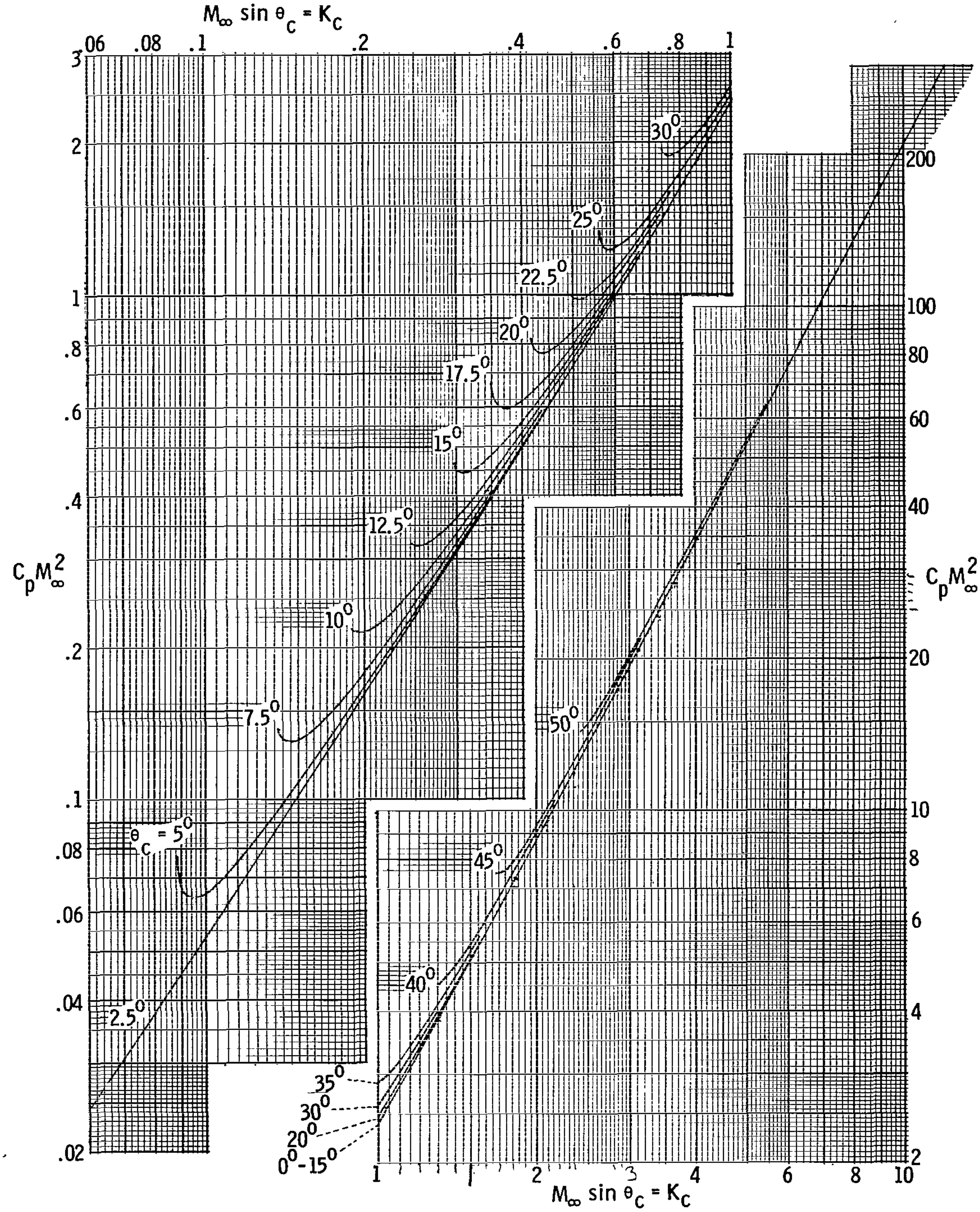
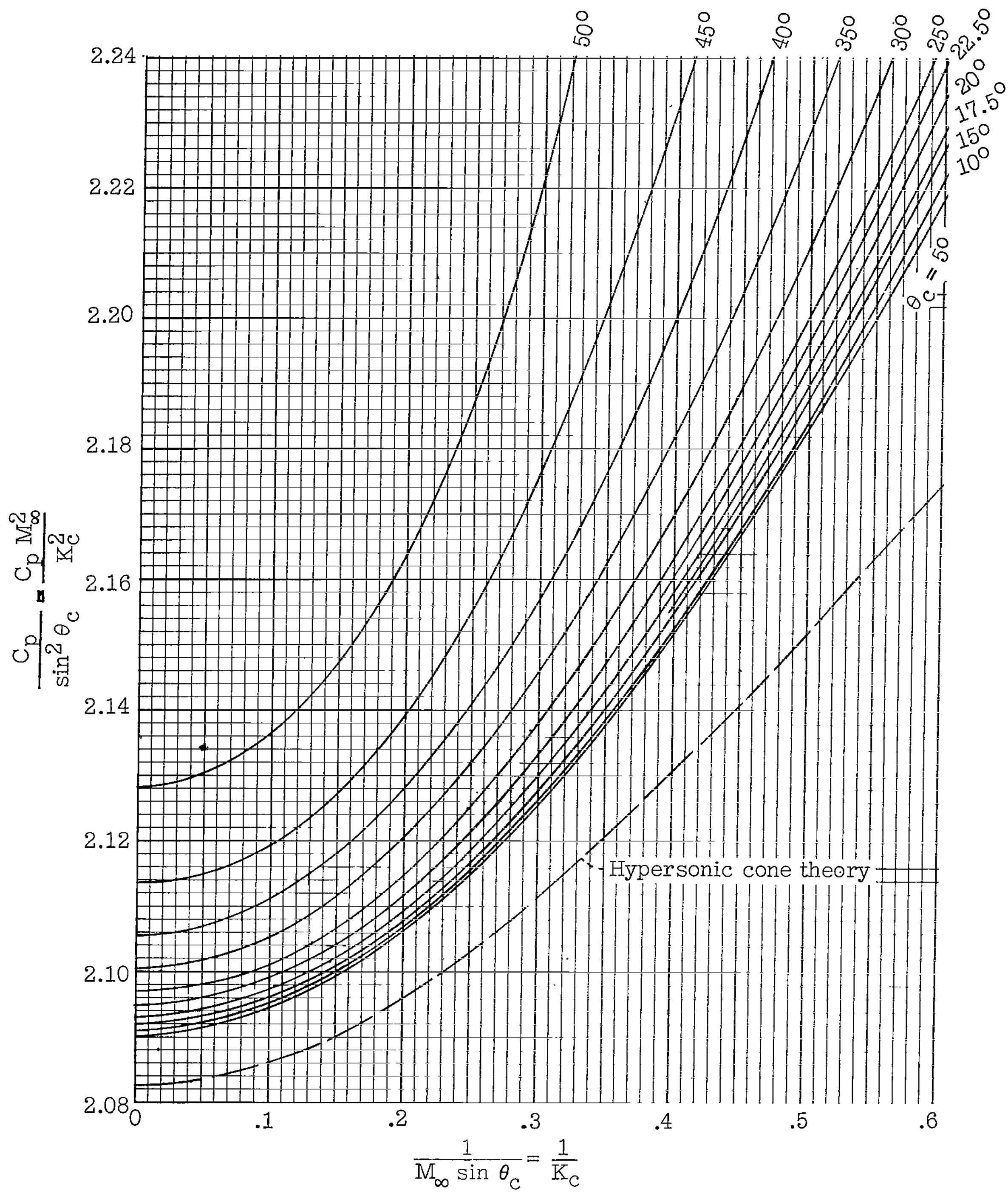
$P_c = P_\infty \left(1 + \frac{1}{2}\gamma c_p M_\infty^2\right)$



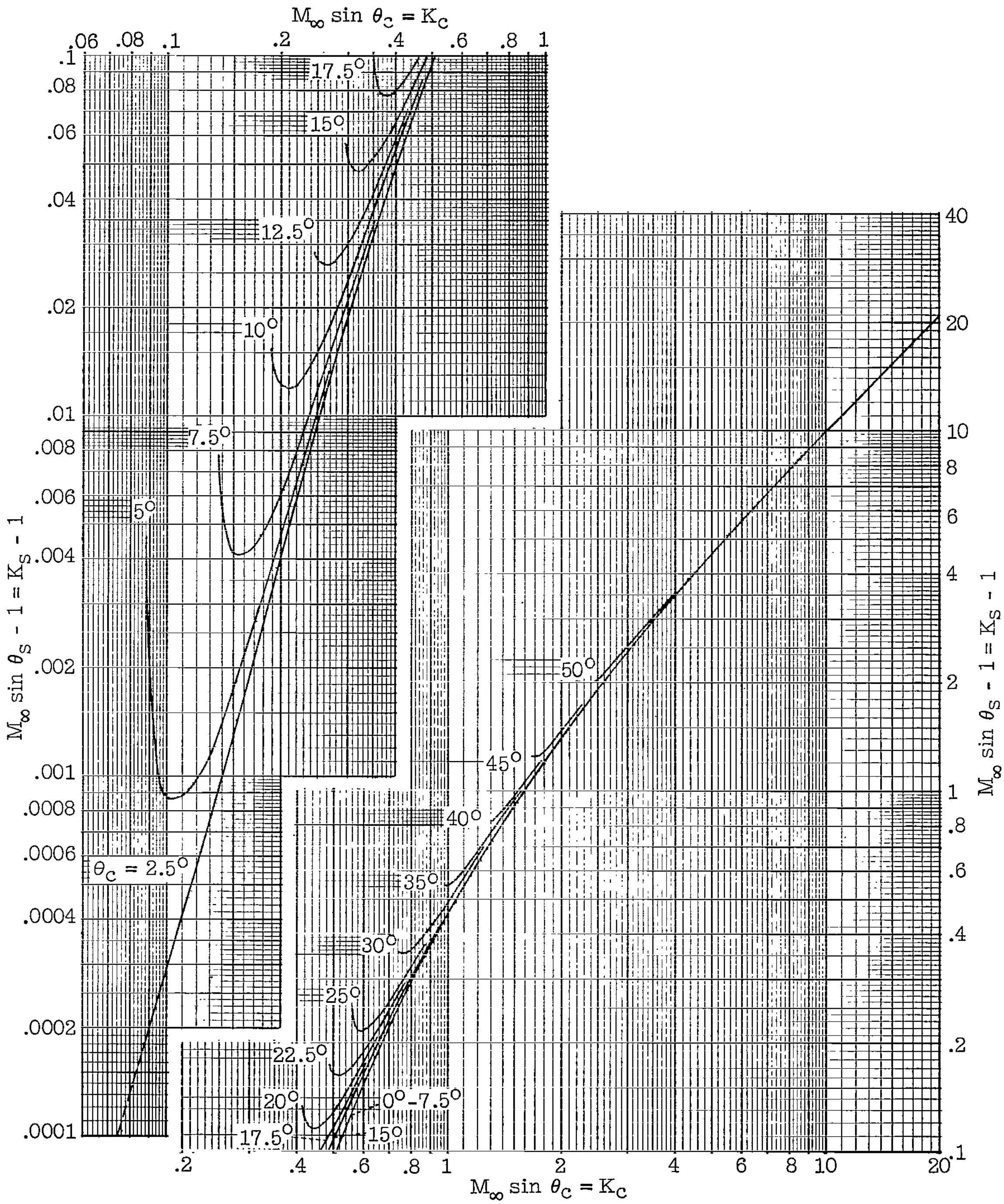
(a) Values of K_c from 0.06 to 12.

Figure 1.- Cone-surface-pressure coefficient in similarity form, applicable to γ = 7/5 and 1.405.



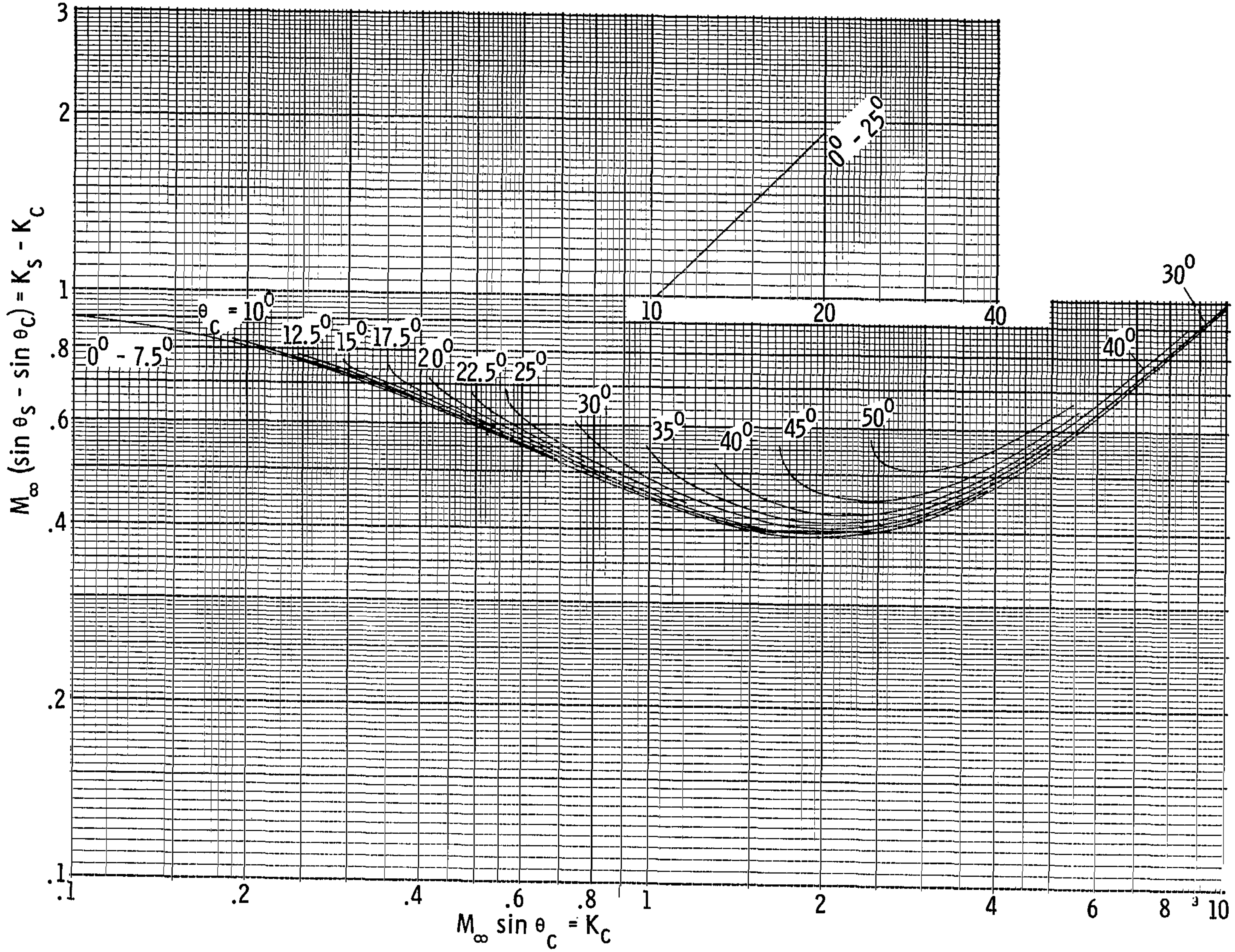
(b) Values of K_c from 1.7 to ∞.

Figure 1.- Concluded.



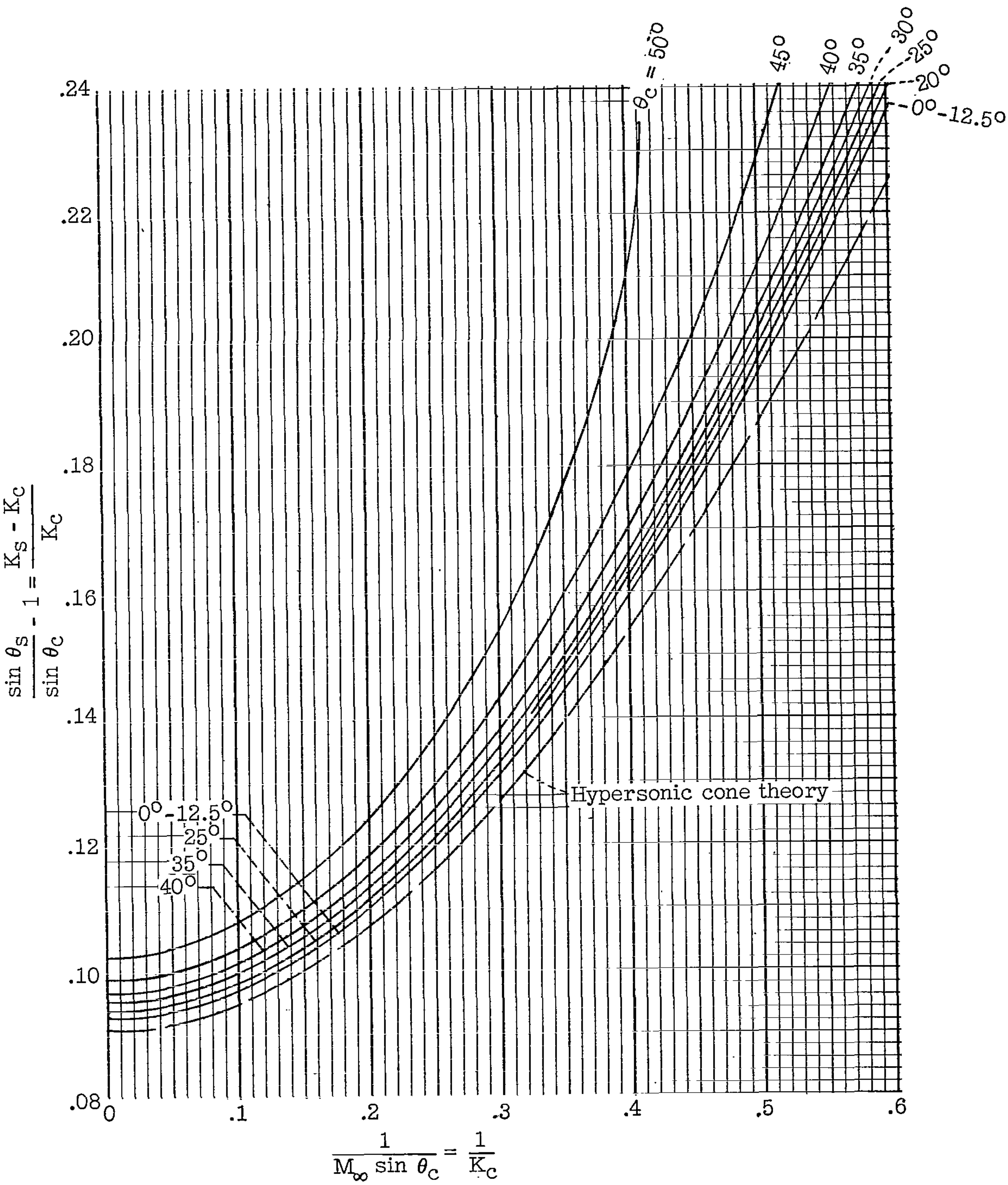
(a) Difference between shock angle and Mach angle, $\gamma = 7/5$ and 1.405.

Figure 2.- Cone shock angle in similarity form.



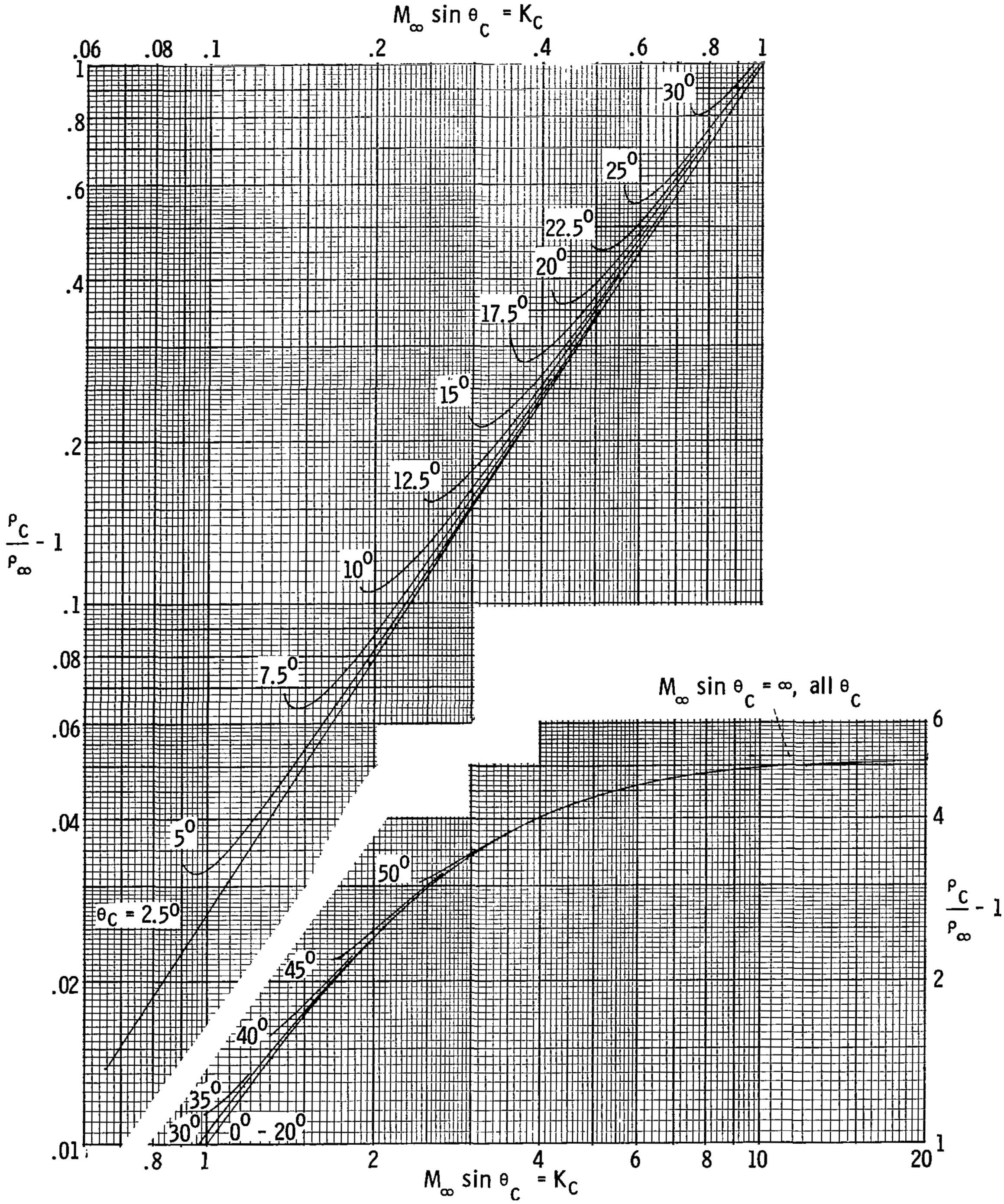
(b) Difference between shock angle and cone angle, $\gamma = 7/5$ and 1.405.

Figure 2.- Continued.



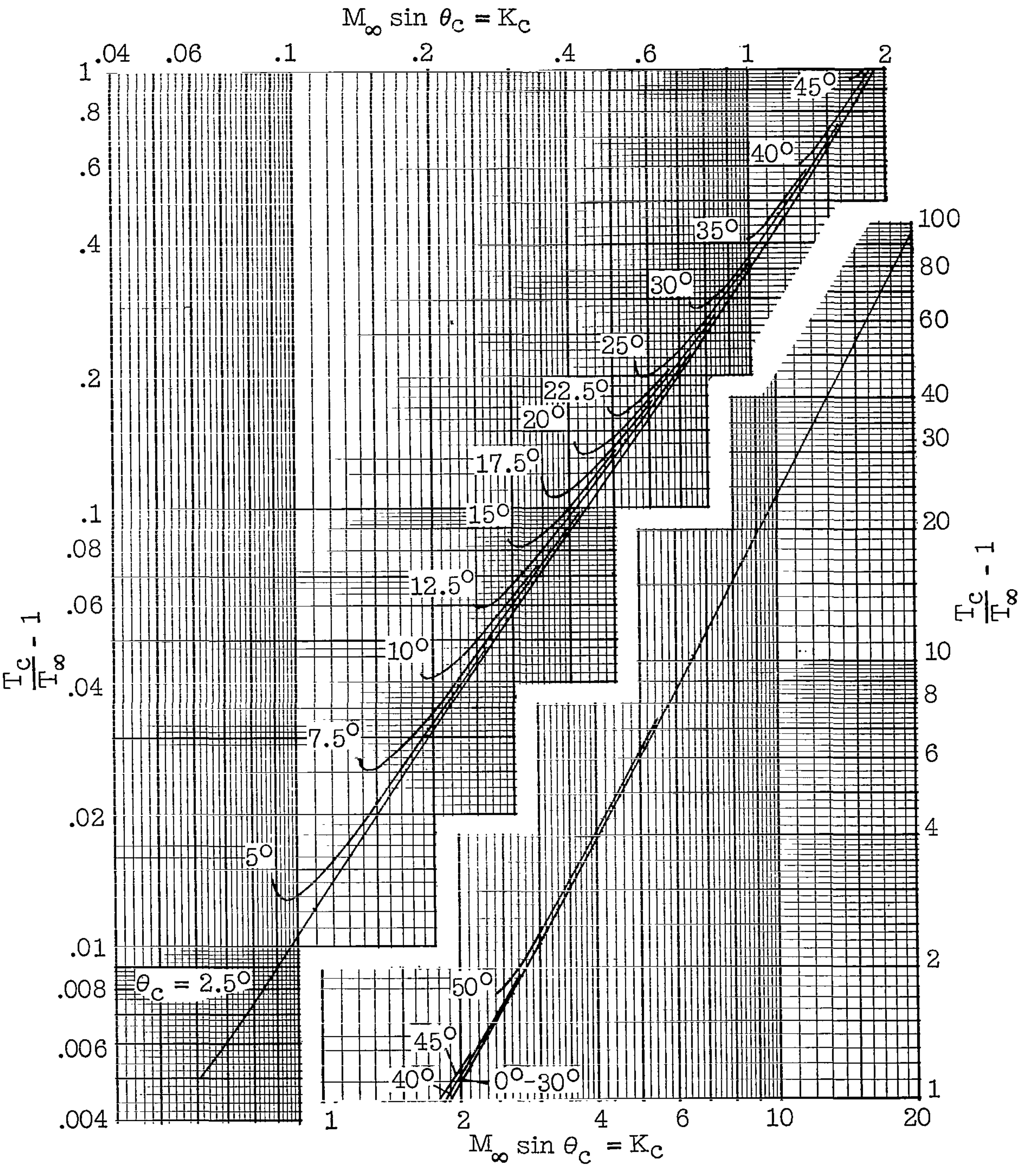
(c) Ratio of shock angle to cone angle, $\gamma = 7/5$ only.

Figure 2.- Concluded.



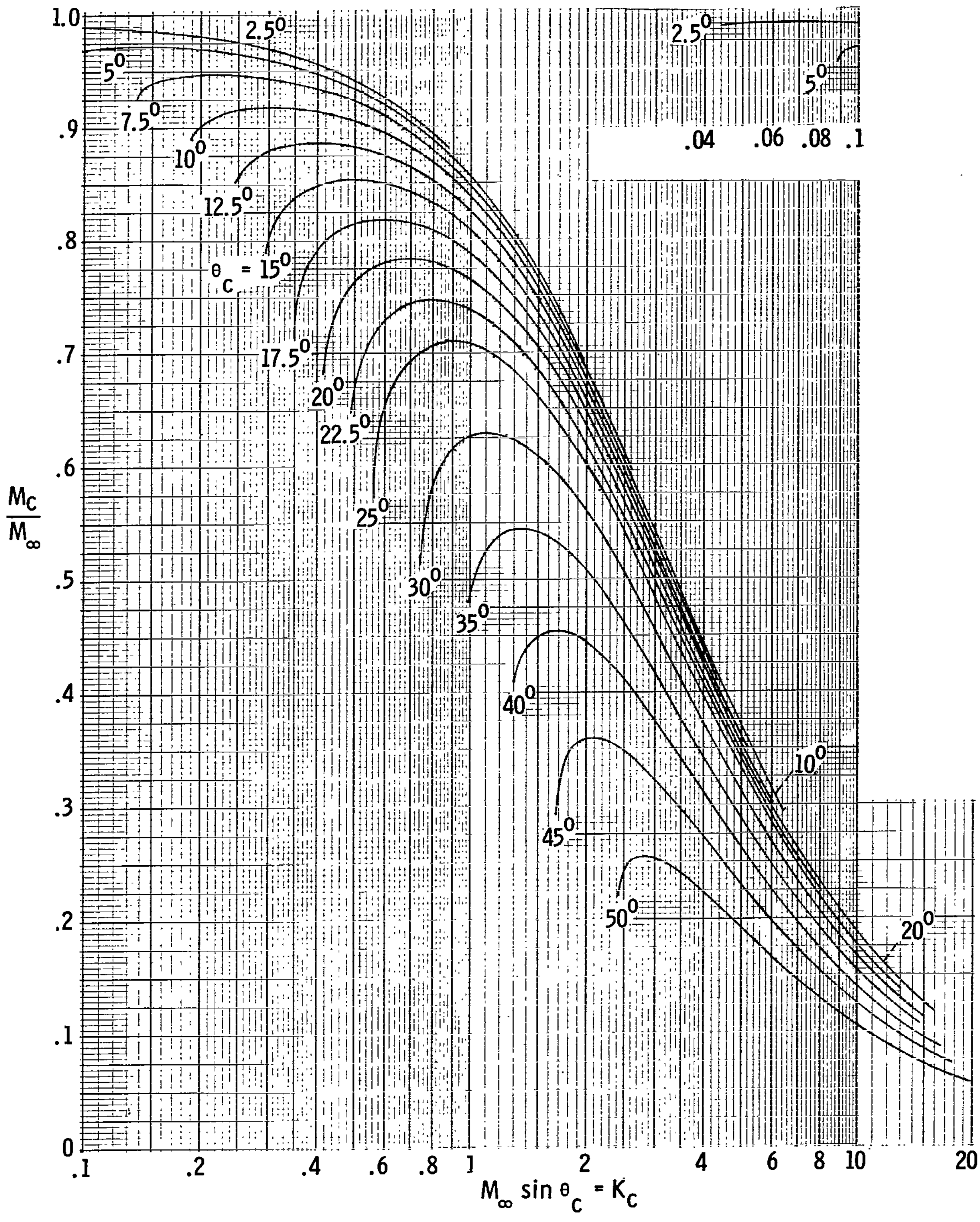
(a) Density ratio.

Figure 3.- Ratio of density, temperature, and Mach number at the cone surface to free-stream values, $\gamma = 7/5$ and 1.405.



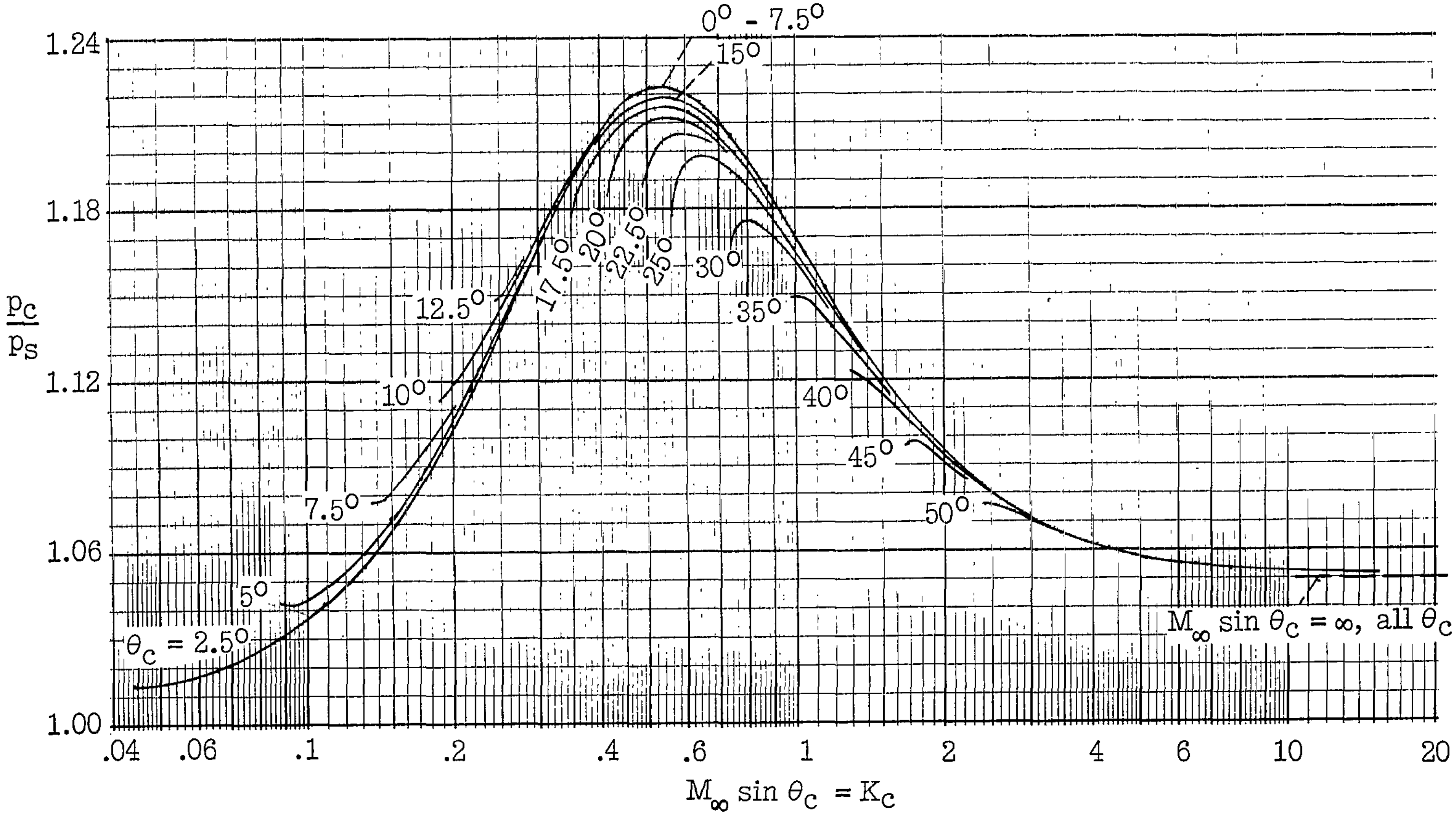
(b) Temperature ratio.

Figure 3.- Continued.



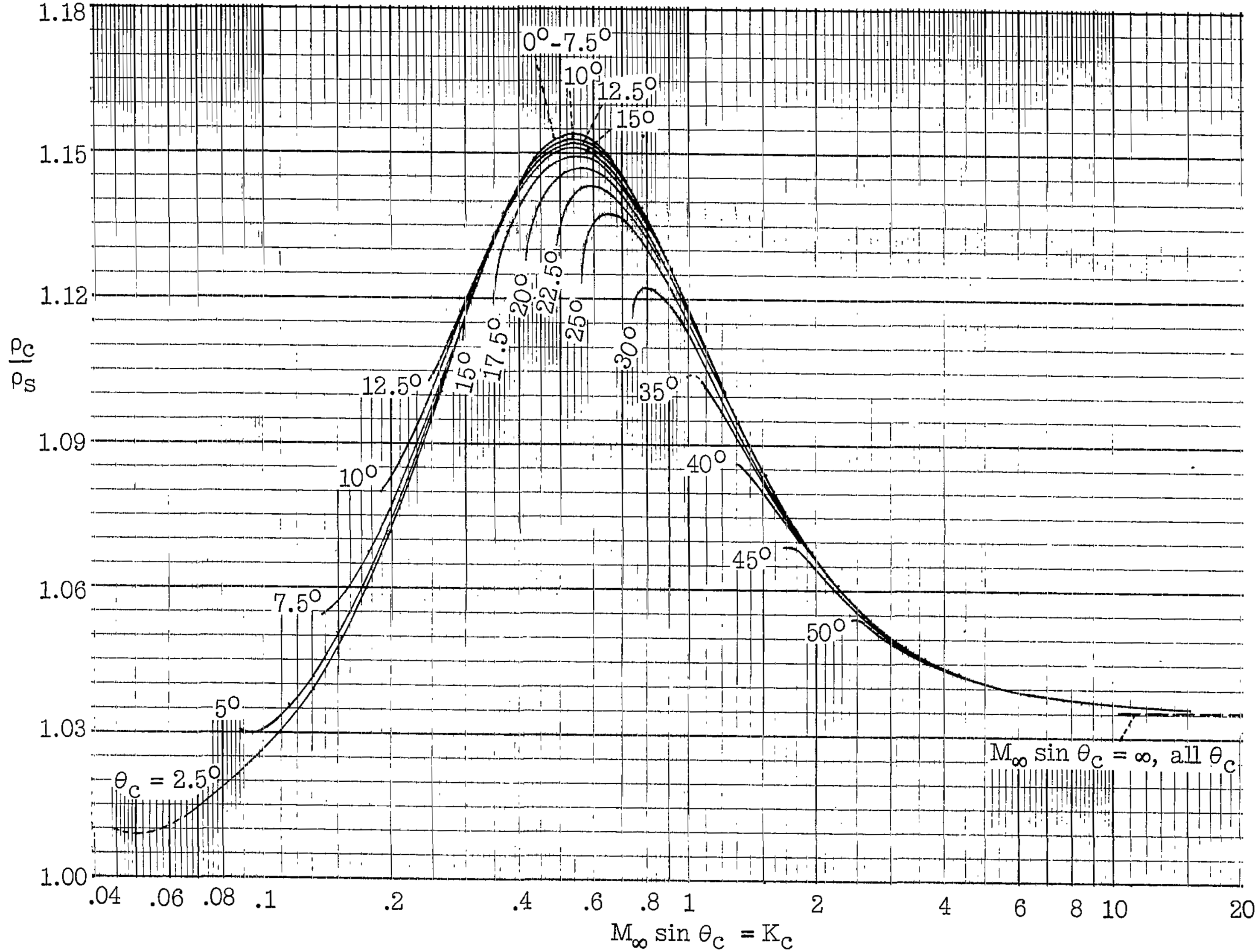
(c) Mach number ratio.

Figure 3.- Concluded.



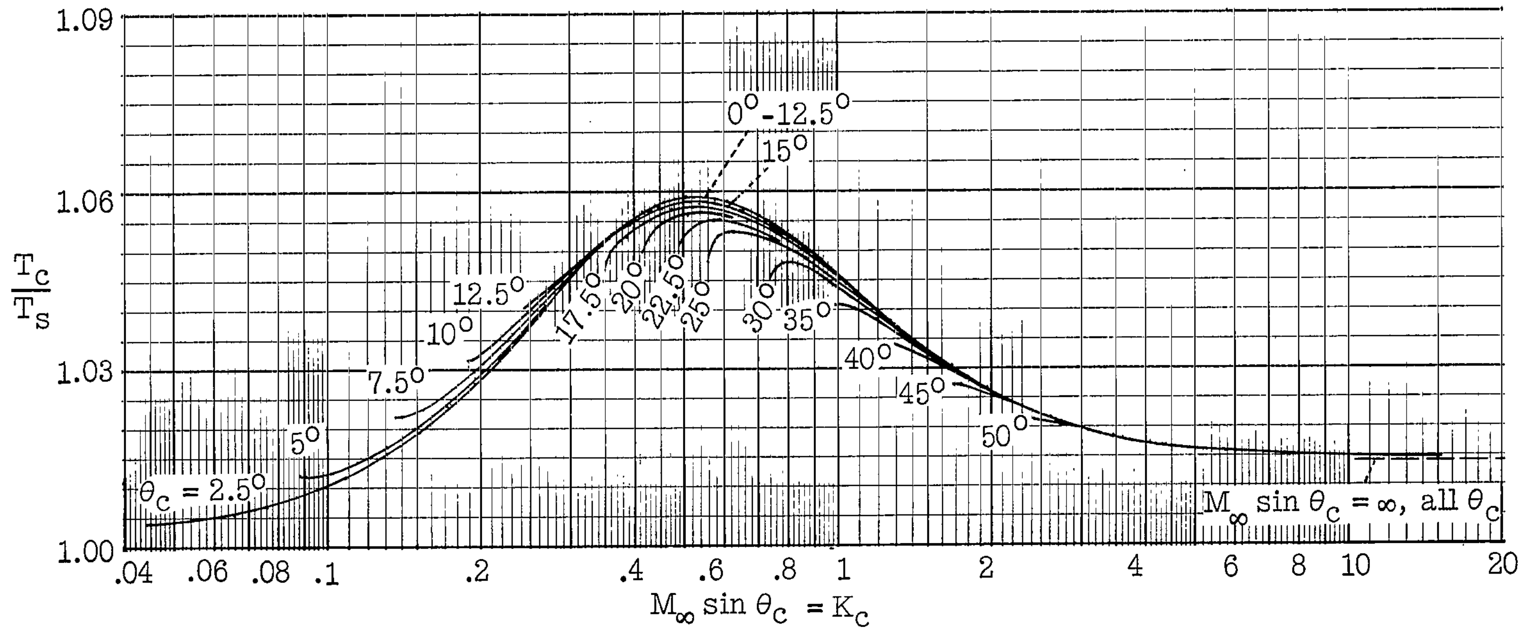
(a) Pressure ratio.

Figure 4.- Ratio of pressure, density, temperature, and Mach number at cone surface to value immediately behind the shock, applicable to $\gamma = 7/5$ and 1.405.



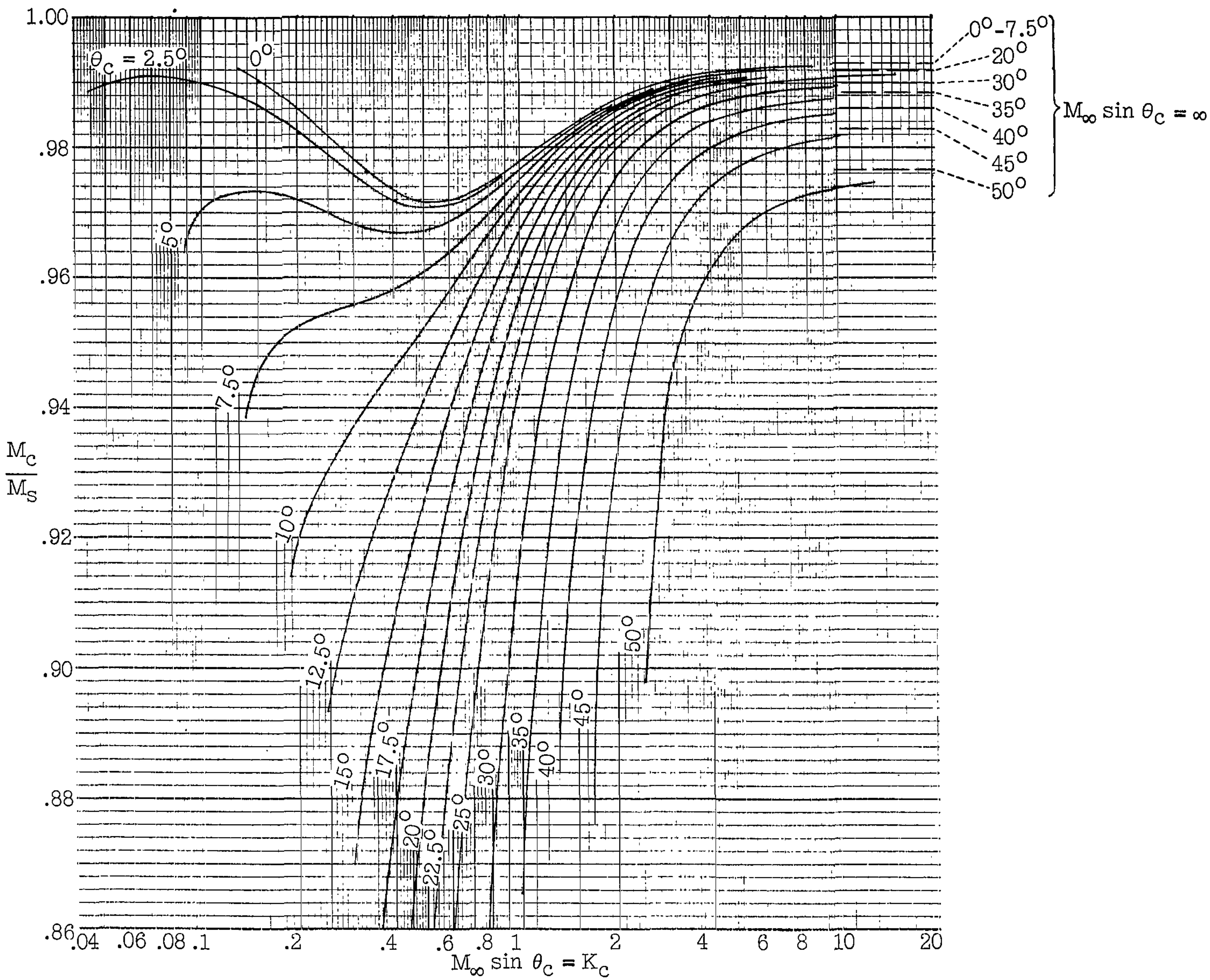
(b) Density ratio.

Figure 4.- Continued.



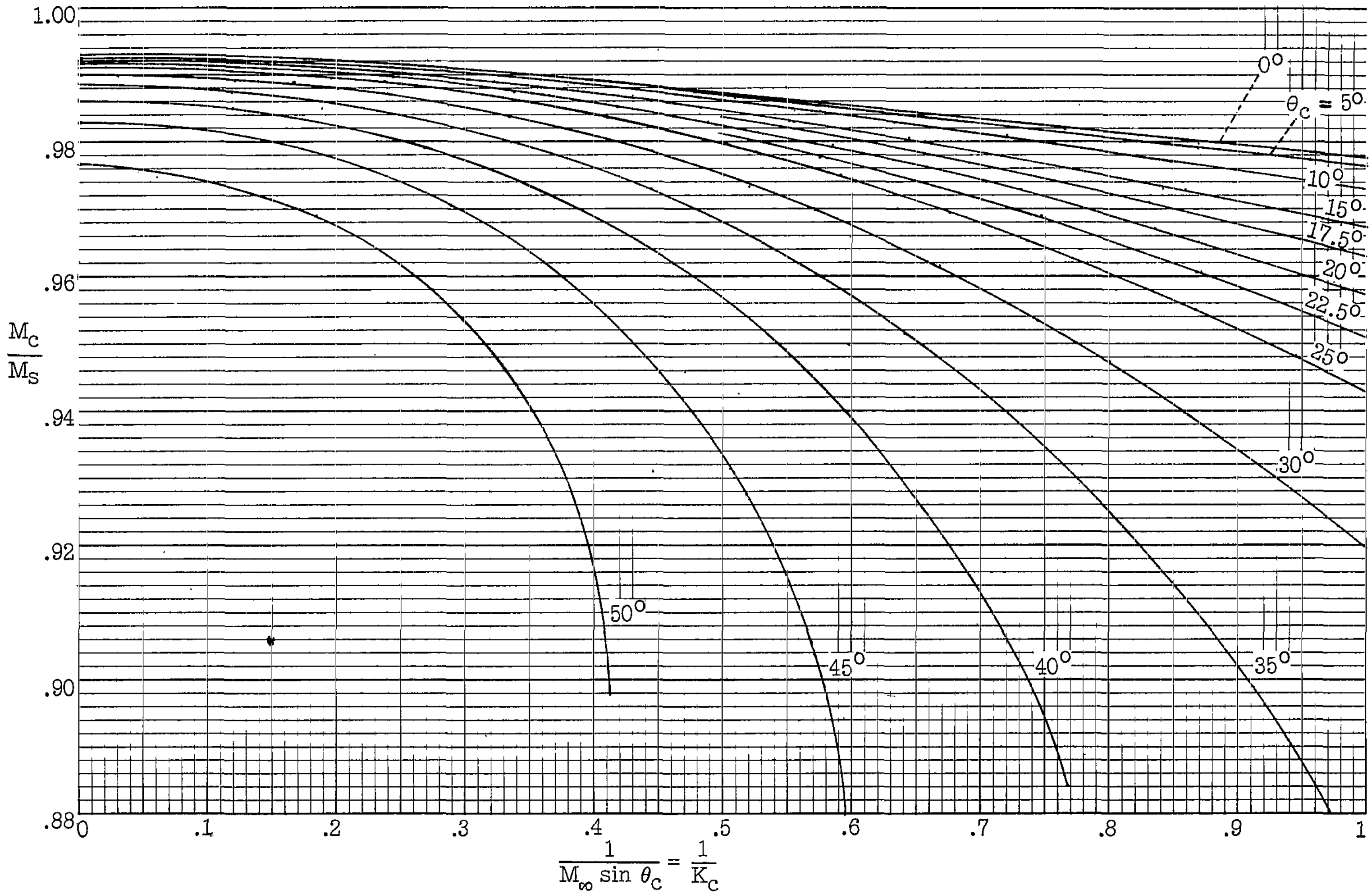
(c) Temperature ratio.

Figure 4.- Continued.



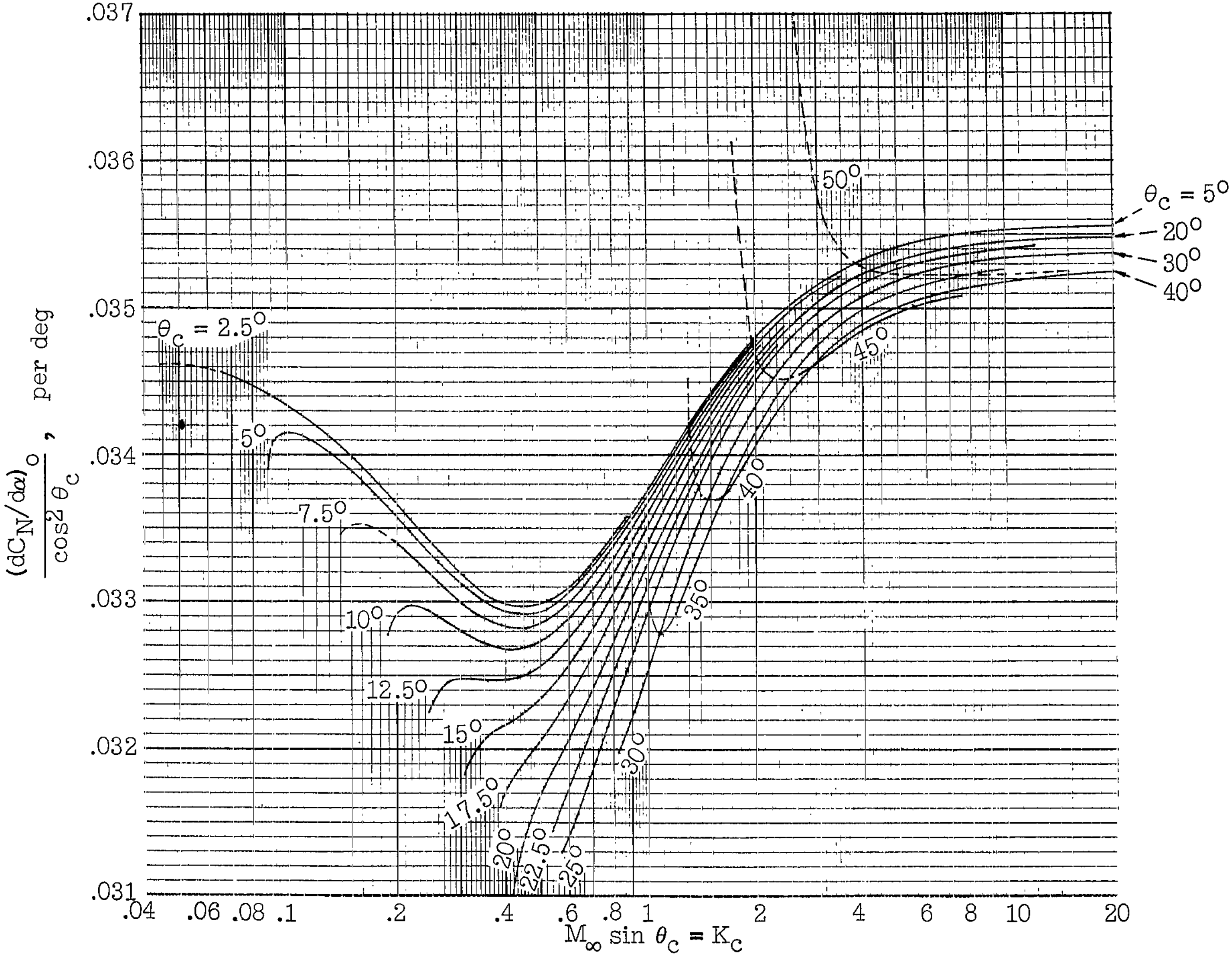
(d) Mach number ratio.

Figure 4.- Continued.



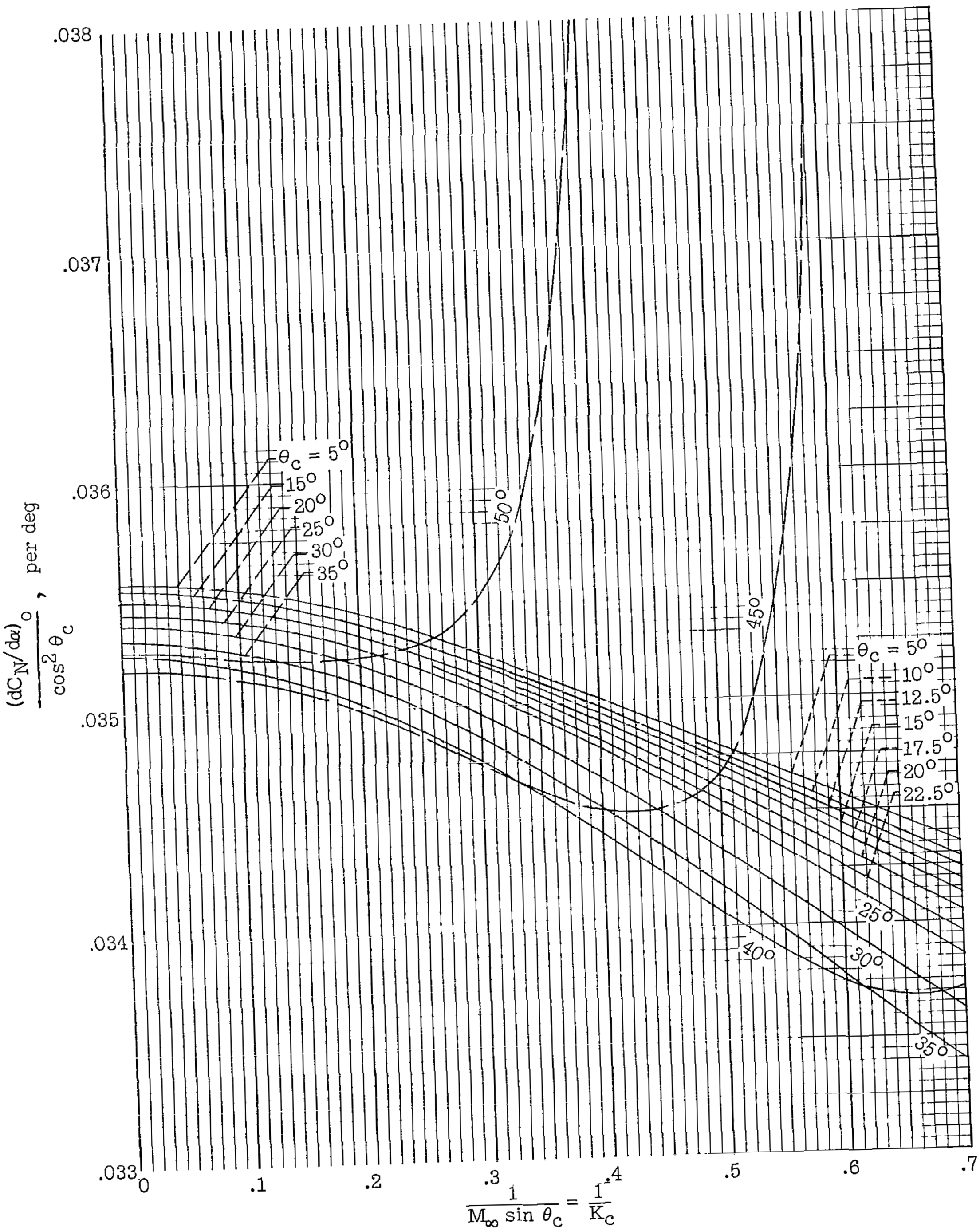
(e) Mach number ratio, K_c from 1 to ∞.

Figure 4.- Concluded.



(a) K_c from 0.04 to 20.

Figure 5.- Initial slope of normal-force-coefficient curve in correlation form, applicable to $\gamma = 7/5$ and 1.405.



(b) K_c from 1.5 to ∞.

Figure 5.- Concluded.

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