Efficiency and Scaling of Constant Inductance Gradient DC Electromagnetic Launchers

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We present efficiency and scaling relationships for dc (i.e., noninduction) constant inductance gradient electromagnetic launchers. We derive expressions for electromagnetic force, efficiency, back-voltage, and kinetic power in terms of electrical circuit parameters. We show that launcher efficiency is a simple function of armature velocity and the launcher's characteristic velocity. The characteristic velocity characterizes the launcher and is the product of two new parameters: the mode constant and launcher constant. Mathematically, the launcher must operate at its characteristic velocity for 50% maximum efficiency. The mode constant reflects the manner in which the launcher is powered and its maximum efficiency. The launcher constant reflects the geometry of the launcher. We consider two modes of operation: constant current and zero exit current operation. We develop the ideal electromagnetic launcher concept and define it as operation at 100% maximum efficiency at all velocities. We also develop the concept of same-scale comparisons, that is, that electromagnetic launcher comparisons should be done with equal bore diameter, launcher length, projectile mass, and velocity. Finally, we present a comparative analysis based on experimental data of same-scale constant gradient electromagnetic launchers for conventional railgun, augmented railgun, and helical gun launchers in terms of the launcher constant, inductance gradient, bore diameter, bore length, system resistance, and armature (i.e., projectile) velocity.

Index Terms—**Coilguns, electromagnetic launching, linear motors, railguns.**

I. Introduction

UNDERSTANDING efficiency and scaling of dc (i.e., noninductive) constant inductance gradient electromagnetic launchers (i.e., EMLs) is important for their continued and successful development. A broad range of applications have been proposed for EMLs including low/high/variable speed, small/large mass, and single-shot/rep-rated systems. The state of knowledge at the present time is insufficient to adequately address the issues associated with these applications. For example, high-speed EML technology may not work for low-speed applications, and vice-versa. High-gradient launchers may have unforeseen benefits in comparison to medium and low-gradient launchers. Despite more than a century of EML research and development [1], factors affecting efficiency and scaling are not well understood. Regardless of the application, good electric-kinetic conversion efficiency and volumetric efficiency (acceleration per amp per volume) are desirable because of the large energy and power involved. It is only natural to use energy conversion and volumetric efficiencies to evaluate and compare different EML geometries. Efficient EML operation reduces pulse power supply size, primary power requirements, switching requirements, physical launcher size, support structure size, and cooling requirements and leads to longer launcher lifetimes.

Constant gradient EMLs include the conventional railgun, augmented railgun, and helical gun illustrated in Fig. 1. The conventional railgun is the simplest EML consisting of two parallel rails and a sliding armature. The augmented railgun has multiple rails connected to increase, or augment, the magnetic field thereby increasing armature force. The helical gun consists of two helical coils, the armature and stator, acting as electromagnets that repel or attract each other. Sliding contacts (i.e., stator and rail brushes in Fig. 1) move with the armature energizing a stator coil section so that armature and stator are always in the optimum force-producing position.

Efficiency and scaling relationships for constant gradient EMLs are derived from basic principles and are related since electromagnetic force is generated in the same manner, specifically through a spatial inductance change known as the inductance gradient. The EML armature usually serves as the projectile and is accelerated by the Lorenz, or $J \times B$, force. In the acceleration process, the armature moves through the magnetic field inducing a voltage according to Faraday's law called the back-voltage. Force, efficiency, and scaling relationships are given in terms of circuit parameters such as inductance gradient, back-voltage, and system resistance and are, therefore, sufficiently general to be applied to any EML geometry. Expressions for the back-voltage and kinetic power are also given and expressed in circuit parameter terms. A comparison of constant gradient EMLs is performed using data from new experiments and data previously reported in the literature.

This investigation reports several new findings and conclusions. The efficiency of constant gradient EMLs is shown to be a function of armature velocity and the launcher's characteristic velocity. The characteristic velocity of the launcher is defined as the product of two parameters developed in this investigation called the launcher constant and the mode constant. The launcher constant reflects the geometry of the launcher. Low-launcher constant geometries approximate an ideal launcher and are efficient. The mode constant reflects the manner in which the EML is operated (or, powered) and its maximum efficiency. Constant current and zero exit current operation modes are investigated. Since the characteristic velocity reflects both the operation mode and geometry of the launcher, it completely characterizes it.

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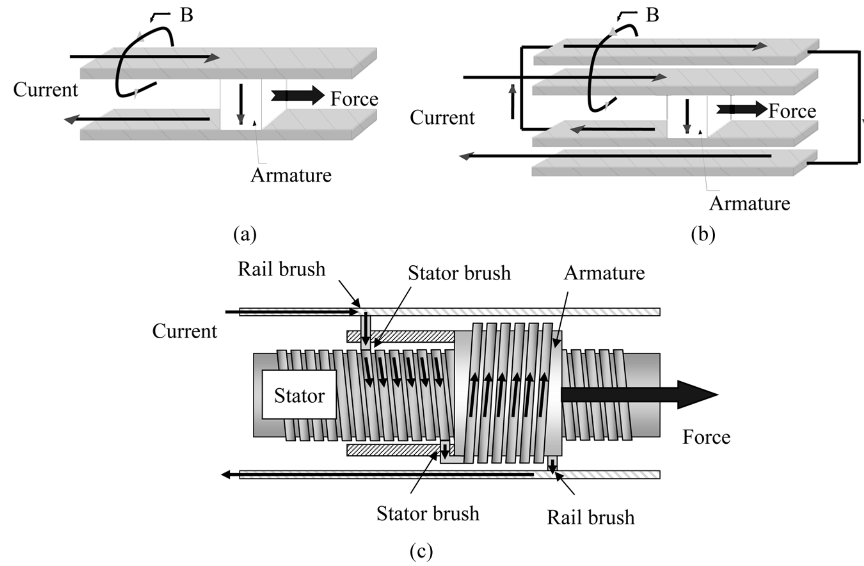

Fig. 1. Constant gradient EML geometry of the (a) conventional railgun, (b) augmented railgun, and (c) helical gun.

Mathematically, the launcher must operate at its characteristic velocity to achieve 50% maximum efficiency. The launcher constant can also be used to characterize the launcher given equal mode constants and operating velocity.

The *ideal* EML is a new concept defined by 100% maximum efficiency operation, regardless of velocity. The concept of *same-scale* comparisons is another new finding allowing different EML geometries to be comparatively analyzed. The same-scale concept states that not only should the bore diameter, bore length, and armature mass be the same when comparing EML geometries, the armature velocity should also be the same. This is because EML efficiency is a function of armature velocity. This investigation also addresses some common misconceptions about EML technology including the notions that high-current EML operation is needed to generate large forces and high-gradient launchers require high-voltage pulsed power supplies. New and recently published experimental results by the authors are used to support the conclusions of this investigation.

II. Theory

A. Electromagnetic Force

Electromagnetic force generated by any electromechanical system is defined as the gradient of the inductively stored energy [3] and expressed mathematically as

$$F \triangleq \nabla W_i \tag{1}$$

where F is the electromagnetic force and W_i is the inductively stored electrical energy stored internally in the launcher. Applying (1) to the conventional and augmented railguns of Figs. 1(a) and (b) yields

$$\begin{aligned} F_{\mathrm{rg}} &= \frac{d}{dx}\left(\frac{1}{2}LI^2\right) \\ &= \frac{1}{2}\frac{dL}{dx}I^2 \\ &= \frac{1}{2}L'I^2 \end{aligned} \tag{2}$$

where F_{rg} is the railgun force, L is the railgun inductance, L' is the railgun inductance gradient, and I is the railgun current. The helical gun geometry of Fig. 1(c) primarily produces force between the armature-stator coil pair. The railgun force term of (2) is present in the helical gun, since the armature-stator coil pair form an equivalent armature, but is many times smaller than the helical gun force and can be ignored. To find the helical gun force, the total equivalent inductance of the armature-stator coil pair is needed and is given by coupled-coil relationship

$$\begin{aligned} L_{\mathrm{eq}} &= (L_a \pm M) + (L_s \pm M) \\ &= L_a + L_s \pm 2M \end{aligned} \tag{3}$$

where L_a is the armature self-inductance, L_s is the stator self-inductance, and M is the mutual inductance between the armature and stator. In (3), the mutual inductance term will be positive for additive magnetic fields and negative for subtractive fields. Fig. 2 illustrates the two helical gun circuit connections possible using standard coupled-coil notation. Differentiating (3) with respect to distance yields the helical gun inductance gradient

$$\begin{aligned} \frac{dL_{\mathrm{eq}}}{dx} &= \frac{d}{dx}(L_a + L_s) \pm \frac{d}{dx}(2M) \\ &= \pm 2\frac{dM}{dx} \\ &= \pm 2M' \end{aligned} \tag{4}$$

where M' is the mutual inductance gradient. Positive M' indicates an attractive electromagnetic force while negative M' indicates a repulsive force. Since there is no change in L_a or L_s with respect to distance, these terms are not involved in force generation. Substituting (4) into (2) yields

$$\begin{aligned} F_{\mathrm{hg}} &= \frac{1}{2}\frac{dL_{\mathrm{eq}}}{dx}I^2 \\ &= M'I^2 \end{aligned} \tag{5}$$

where F_{hg} is the helical gun force [5].

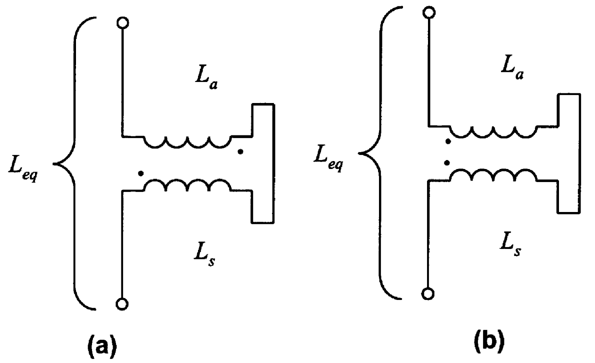

Fig. 2. The two electrical connections possible with helical gun geometry showing (a) additive magnetic fields and (b) subtractive magnetic fields.

B. Kinetic Power

The three geometries of Fig. 1 have current-carrying armatures moving through a magnetic field. When a conductor moves through a magnetic field, a voltage is induced at its terminals according to Faraday's law as

$$V_{\text{ind}} \triangleq -\frac{d\Psi}{dt} \tag{6}$$

where V_{ind} is the induced voltage and Ψ is the total flux linkage. Lenz's law gives the induced armature voltage polarity which can be safely ignored in this investigation. For the most general treatment, Faraday's law is expressed in terms of electric circuit parameters. Specific EML geometry information can be inserted at a later stage as needed. The induced armature voltage becomes

$$\begin{aligned} V_{\text{ind}} &= \frac{d(LI)}{dt} \\ &= L\frac{dI}{dt} + I\frac{dL}{dt} \\ &= V_e + V_b. \end{aligned} \tag{7}$$

The first term of (7), V_e, is the usual inductor voltage produced when charging the inductor to a given energy state. The second term of (7), V_b, is the so-called back-voltage produced when performing mechanical work on the inductor, e.g., changing its shape or location in space.

The product of the back voltage and armature current is termed the kinetic power and represents the electrical power used to produce motion. The kinetic power for conventional and augmented railguns is found by multiplying the second term of (5) and the armature current given as

$$\begin{aligned} P_{\text{krg}} &= IV_b \\ &= I\left(I\frac{dL}{dt}\right) \\ &= I\left(I\frac{dL}{dx}\frac{dx}{dt}\right) \\ &= I^2L'v \end{aligned} \tag{8}$$

where P_{krg} is the railgun kinetic power and v is the armature velocity. Substituting the helical gun inductance gradient term of (4) into (8) yields the kinetic power expression for the helical gun

$$\begin{aligned} P_{\text{khg}} &= I\left(I\frac{dL_{\text{eq}}}{dx}\frac{dx}{dt}\right) \\ &= I^2L'_{\text{eq}}v \\ &= 2I^2M'v \end{aligned} \tag{9}$$

where P_{khg} is the helical gun kinetic power.

C. Efficiency

The constant gradient EML electric-kinetic conversion efficiency is defined as the ratio of the output energy and the total input energy given as

$$\eta \triangleq \frac{W_k}{W_k + W_r + W_i + W_c + W_f} \tag{10}$$

where η is the efficiency, W_k is the kinetic energy, W_r is the resistive energy losses, W_i is the inductive energy stored or lost to commutation (all other inductive energy storage is assumed zero), W_c is the contact energy losses, and W_f is the friction energy losses. High efficiency results if the kinetic energy is much greater than the sum of the resistive, inductive, contact, and frictional energy terms. Assuming efficient sliding contacts and negligible frictional losses, (10) can be further simplified to

$$\begin{aligned} \eta &= \frac{W_k}{W_k + W_r + W_i} \\ &= \frac{1}{1 + \frac{W_r}{W_k} + \frac{W_i}{W_k}}. \end{aligned} \tag{11}$$

In applying (11) to the EMLs of Fig. 1, consideration is given to the manner in which the launcher is operated since that determines its energy state and, subsequently, the substitutions for the various terms in (11). Two modes of operating the EML are considered. In the constant current (i.e., CC) mode, current is constant during the entire acceleration event interrupted only when the armature leaves the launcher. In the zero exit current (i.e., ZC) mode, current is increased to a given level but is zero as the armature exits the launcher. The current can decay to zero in a natural manner, as prescribed by the electrical circuit, or it can be forced to zero with an external circuit [2]. Mechanical methods physically interrupting current flow are not acceptable in the present context. The reason for this pertains to inductive energy storage in the launcher and will be detailed in the following section.

1) Constant Current Operation: The CC operation mode is applied to the conventional and augmented railgun. With constant I, the railgun force of (1) is integrated with respect to distance yielding the railgun armature kinetic energy

$$\begin{aligned} W_{\text{krg}} &= \int F_{\text{rg}}dx \\ &= \int \frac{1}{2}\frac{dL}{dx}I^2dx \\ &= \frac{1}{2}LI^2 \\ &= W_{\text{irg}} \end{aligned} \tag{12}$$

where W_{krg} is the railgun kinetic energy and W_{irg} is the railgun inductive energy. Equation (12) shows that the railgun armature kinetic energy is equal to the inductively stored energy. Therefore, with $W_{\mathrm{krg}} = W_{\mathrm{irg}}$, (11) can be further reduced to

$$\eta_{\mathrm{rgcc}} = \frac{1}{2 + \frac{W_{\mathrm{rrg}}}{W_{\mathrm{krg}}}} \tag{13}$$

where η_{rgcc} is the railgun efficiency in CC mode and W_{rrg} is the railgun resistive losses. Another expression for the railgun kinetic energy is needed for (13) and can be obtained by integrating the kinetic power in (8) with respect to time. In CC mode, the velocity will increase linearly in time. Assuming the inductance gradient is constant, the result of this integration is given by

$$\begin{aligned} W_{\mathrm{krg}} &= \int f_{\mathrm{rg}} P_{\mathrm{krg}}\, dt \\ &= f_{\mathrm{rg}} \int (I^2 L' v)\, dt \\ &= f_{\mathrm{rg}} \frac{1}{2} I^2 L' v_{\max} \tau \end{aligned} \tag{14}$$

where W_{krg} is the railgun kinetic energy, f_{rg} is the fraction of the kinetic power used to accelerate the railgun armature, $v_{\max}$ is the maximum armature velocity, and τ is the pulse length. The other fraction of the kinetic power is used to charge the railgun inductance. This statement is true because the power used to charge the inductor is included in the product of IV_b. The power is not in the product of IV_e since V_e is zero.

Rearranging the terms of (14) and substituting (2) yields

$$\begin{aligned} W_{\mathrm{krg}} &= \left(\frac{1}{2} I^2 L'\right) f_{\mathrm{rg}} v_{\max} \tau \\ &= F_{\mathrm{rg}} f_{\mathrm{rg}} v_{\max} \tau. \end{aligned} \tag{15}$$

For consistency, the condition $f_{\mathrm{rg}} = 1/2$ must be true, so that (15) will reduce to the expected result given by

$$\begin{aligned} W_{\mathrm{krg}} &= F_{\mathrm{rg}} \frac{1}{2} v_{\max} \tau \\ &= F_{\mathrm{rg}} v_{\mathrm{avg}} \tau \\ &= F_{\mathrm{rg}} \Delta x \end{aligned} \tag{16}$$

where Δx is the length of the launcher. Since $f_{\mathrm{rg}} = 1/2$, one-half of W_{krg} is converted to motion and one-half is stored inductively, as is already known from previous statements and (12).

A suitable expression for the resistive energy term in (11) is given by the definition

$$W_r \triangleq \int I^2 R dt \tag{17}$$

where R is the total system resistance. Assuming that R is also constant, then (17) becomes

$$W_{\mathrm{rcc}} = I^2 R \tau \tag{18}$$

where W_{rcc} is the resistive energy losses. Constant system resistance is not true in practice because of joule heating and high-frequency skin effects. An average system resistance can be used in these cases. Equations (15) and (18) are substituted into (13) yielding railgun efficiency

$$\begin{aligned} \eta_{\mathrm{rgcc}} &= \frac{1}{2 + \frac{4 I^2 R \tau}{I^2 L' v_{\max} \tau}} \\ &= \left(\frac{1}{2}\right) \frac{1}{1 + \frac{2R}{L' v_{\max}}}. \end{aligned} \tag{19}$$

The helical gun is the next EML geometry to be analyzed and suitable expressions are sought for the terms of (11). In CC mode, the helical gun force of (5) is integrated with respect to distance yielding the kinetic energy relationship of

$$\begin{aligned} W_{\mathrm{khg}} &= \int F_{\mathrm{hg}} dx \\ &= \int M' I^2 dx \\ &= M I^2 \\ &= W_{\mathrm{ihg}} \end{aligned} \tag{20}$$

where W_{khg} is the helical gun kinetic energy and W_{ihg} is the helical gun inductive energy lost during acceleration. Furthermore, assuming $L_{\mathrm{eq}} \ll M$, there is no inductive energy stored since the helical gun uses only a short length of stator coil. The helical gun efficiency expression, therefore, has a form similar to the railgun efficiency of (13), namely

$$\eta_{\mathrm{hgcc}} = \frac{1}{2 + \frac{W_{\mathrm{rhg}}}{W_{\mathrm{khg}}}} \tag{21}$$

where η_{hgcc} is the helical gun efficiency in CC mode and W_{rhg} is the helical gun resistive losses. Proceeding as was done in (14), another helical gun kinetic energy expression can be found as

$$\begin{aligned} W_{\mathrm{khg}} &= f_{\mathrm{hg}} \int P_{\mathrm{khg}}\, dt \\ &= f_{\mathrm{hg}} \int (2 M' I^2 v)\, dt \\ &= (M' I^2) f_{\mathrm{hg}} v_{\max} \tau \\ &= F_{\mathrm{hg}} f_{\mathrm{hg}} v_{\max} \tau \end{aligned} \tag{22}$$

where f_{hg} is the fraction of the kinetic power used to accelerate the helical gun armature. As before, the condition $f_{\mathrm{hg}} = 1/2$ must be true, so that (22) will reduce to $F_{\mathrm{hg}} \Delta x$. One-half of W_{khg} is converted to motion and one-half is lost to commutation. Substituting (18) and (22) into (21) and rearranging terms yields the helical gun efficiency in CC mode as

$$\eta_{\mathrm{hgcc}} = \left(\frac{1}{2}\right) \frac{1}{1 + \frac{R}{M' v_{\max}}}. \tag{23}$$

2) Zero Exit Current Operation: The ZC operation mode simplifies some of the previous analysis since there will be no inductive energy storage in the launcher at armature exit. If the current decays to zero naturally, as prescribed by the L/R time constant of the system, the inductive energy will be used toward acceleration. If the current is forced to zero with the aid of an energy recovery circuit [2], the inductively stored energy is removed from the system and the efficiency equation. In both cases, $W_i = 0$ which reduces (11) to

$$\eta_{\rm zc} = \frac{1}{1 + \frac{W_r}{W_k}} \tag{24}$$

where $\eta_{\rm zc}$ is the efficiency in ZC mode. The launcher velocity is not linear since the current is not constant making direct integration in (14) and (22) impossible. In this mode of operation, we start with the familiar kinetic energy expression

$$W_k = \frac{1}{2}mv^2. \tag{25}$$

The momentum of the conventional and augmented railgun armature is given by

$$\begin{aligned} p_{\rm rg} &= mv \\ &= \int F_{\rm rg} dt \\ &= \frac{1}{2}L' \int I^2 dt \end{aligned} \tag{26}$$

where m is the railgun armature mass and $p_{\rm rg}$ is its linear momentum. Substituting (26) into (25) yields the kinetic energy expression

$$\begin{aligned} W_{\rm krg} &= \frac{1}{2}(mv)v \\ &= \frac{1}{4}L'v \int I^2 dt. \end{aligned} \tag{27}$$

The resistive energy definition of (17) with constant system resistance becomes

$$W_{\rm rzc} = R \int I^2 dt \tag{28}$$

where $W_{\rm rzc}$ is the resistive energy in ZC mode. Substituting (28) and (27) into (24) yields the conventional and augmented railgun efficiency

$$\begin{aligned} \eta_{\rm rgzc} &= \frac{1}{1 + \frac{R\int I^2 dt}{\frac{1}{4}L' v_{\max} \int I^2 dt}} \\ &= \frac{1}{1 + \frac{4R}{L' v_{\max}}} \end{aligned} \tag{29}$$

where $\eta_{\rm rgzc}$ is the railgun efficiency in ZC mode. The substitution $v = v_{\max}$ is made since maximum efficiency is the only case of interest.

The efficiency for the helical gun EML operating in ZC mode is found by substituting the term $L' = L'_{\rm eq} = 2M'$ in (29) to yield the final helical gun efficiency given as

$$\eta_{\rm hgzc} = \frac{1}{1 + \frac{2R}{M' v_{\max}}}. \tag{30}$$

III. Discussion

Comparing (2) and (5), the electromagnetic force is proportional to the square of the armature current. The force is also noted to be proportional to the inductance gradient of the EML geometry. Greater force can be produced by increasing the current a factor of 2, for example, than by increasing the inductance gradient a similar amount. Helical gun launchers have an additional factor of 2 in their force expression due to mutual inductance in comparison to railgun launchers.

The kinetic power expression given by (8) and (9) is the rate at which energy is delivered to the armature to produce acceleration and is the product of the armature current and the back-voltage. Like the mechanical force, the kinetic power is proportional to the square of the armature current. Unlike the electromagnetic force, however, the kinetic power is proportional to the armature velocity. The back voltage increases as the armature accelerates. As in the force expressions, helical gun launcher geometries have an additional factor of 2 in their kinetic power expression in comparison to railgun launchers.

The electromagnetic force and kinetic power I-squared dependency might lead one to conclude that high-current EML operation is needed for large force production. While high current will certainly produce large kinetic power and force, it will simultaneously produce large resistive power loss. Equations (19) and (29) clearly show that any increase in kinetic energy resulting from increased EML current is proportionally offset by increased resistive losses. High-current EML operation should be avoided for high-efficiency operation. Section IV will show that large electromagnetic forces can be generated with low current.

Examination of the railgun efficiency of (19) and (29) and the helical gun efficiency of (23) and (30) show that efficiency for these devices can be generalized to the expression

$$\begin{aligned} \eta &= \left(\frac{\mu}{4}\right) \frac{1}{1 + \frac{\mu\lambda}{v_{\max}}} \\ &= \eta_{\max} \frac{1}{1 + \frac{\mu\lambda}{v_{\max}}} \end{aligned} \tag{31}$$

where μ is a term reflecting the mode of operation ($\mu = \mu_{\rm cc} = 2$ for CC mode and $\mu = \mu_{\rm zc} = 4$ for ZC mode), λ is a term reflecting the launcher's geometry, and $\eta_{\max} = \mu/4$ is the maximum efficiency. In this investigation, $\mu$is termed the *mode constant*, and λ is termed the *launcher constant*. The launcher constant is the ratio of the system resistance and the inductance gradient. For conventional and augmented railguns, the launcher constant is given as

$$\lambda_{\rm rg} = \frac{R}{L'} \tag{32}$$

whereas for helical guns the launcher constant is given as

$$\lambda_{\rm hg} = \frac{R}{2M'}. \tag{33}$$

Equation (31) shows that efficiency is clearly a function of the armature velocity. Although velocity-dependent EML efficiency will be experimentally verified in the following section, it should not be surprising since rotational dc motors are known to be inefficient in the start-up process [3]. The dc rotational motor has almost zero back-voltage (i.e., a short circuit) in the start-up phase with almost no electrical power being used to produce motion. As the motor gains speed, the

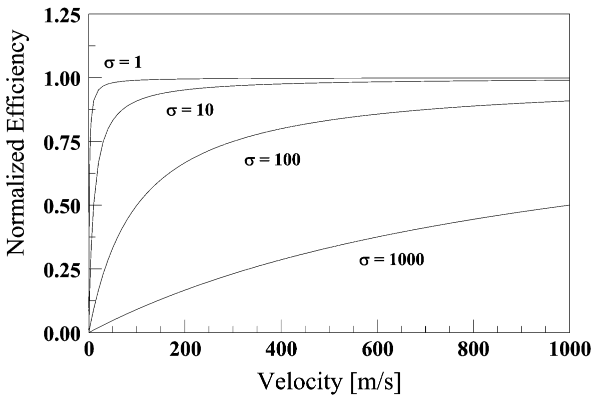

Fig. 3. Normalized efficiency versus velocity for various characteristic velocities.

back-voltage increases, more power is used for motion, and the motor approaches its steady-state efficiency. A similar scenario occurs for the EMLs in this investigation.

There are two limiting cases of efficiency in (31) with respect to velocity, specifically $v = 0$ and $v = \infty$. At low velocity, EMLs are inefficient while at high velocity, EMLs approach maximum efficiency. The EML back-voltage and kinetic energy are low at low velocity with little electrical power being used to produce motion. The resistive energy term dominates in (13) and (24) producing low efficiency. At high velocity, however, the back-voltage and kinetic power are high with a larger fraction of the electrical energy used to produce motion. The resistive energy term is negligible in comparison to the kinetic energy term and the efficiency is high.

Low velocity and *high velocity* are relative to the product of the mode constant and launcher constant. Normalizing (31) with respect to $\eta_{\max}$ yields the normalized EML efficiency of

$$\frac{\eta}{\eta_{\max}} = \frac{1}{1+\frac{\mu\lambda}{v_{\max}}} = \frac{1}{1+\frac{\sigma}{v_{\max}}} \quad (34)$$

where $\sigma = \mu\lambda$ is termed the *characteristic velocity*. If $v_{\max} \ll \sigma$, the velocity is considered low and the efficiency is low. If $v_{\max} \gg \sigma$, the velocity is considered high and the efficiency is high. When $v_{\max} = \sigma$, the launcher operates at 50% maximum theoretical efficiency.

Low-σ geometries are synonymous with high efficiency. Fig. 3 plots the normalized efficiency of (34) versus velocity for $\sigma = 1, 10, 100$, and 1000. As can be seen in that figure, low-σ launchers approach maximum efficiency more quickly than high σ launchers. The characteristic velocity can, therefore, be used to characterize the EML. The launcher constant λ can also be used to characterize an EML if one assumes a fixed operating mode (i.e., CC or ZC) and armature velocity.

The Fig. 3 data also suggests that an *ideal launcher* is one that operates at 100% maximum efficiency, regardless of velocity. For example, a railgun or helical gun operating in CC mode at 50% efficiency would be considered an *ideal railgun* or an *ideal helical gun*. It would be unreasonable to define the ideal launcher as one that achieves 100% efficiency if the launcher is not operated in a mode that can attain 100% efficiency. Although the ideal launcher may be difficult to achieve in practice, the Fig. 3 case with $\sigma = 1$ is very close to ideal and is approximately 90% normalized efficient for $v \geq 10$ m/s. In comparison, a launcher with $\sigma = 1000$ must operate at 10 000 m/s for 90% normalized efficiency. A low-σ EML geometry approximates the ideal launcher.

The launcher constant is also a scaling factor reflecting the benefits derived when changes are made to a particular EML geometry. Specific EML geometry information can now be substituted in (31) or (34). Obviously, a low-λ geometry is desired and is achieved by lowering the system resistance or increasing the inductance gradient.

Inductive energy use in constant gradient EMLs is determined by (12) and (20) that state regardless of operation mode, the EML will inductively store (or, consume) an energy equal to the kinetic energy of the projectile. If the EML is operated in the CC mode, then the stored energy is ultimately lost (either resistively as heat or acoustically as in arc blast when the projectile exits the launcher). If the EML is operated in ZC mode, then the stored energy can be used to accelerate the projectile or can be removed, or recovered, from the system.

A final point to be made in this section regards the process by which different EML geometries are compared. From (34), the efficiency of a constant gradient EML is a function of both the armature velocity and the launcher's characteristic velocity. If the operation mode and armature velocity is fixed and the geometry has equal inductance gradient and system resistance, the helical gun will be the most efficient geometry simply due to the additional factor of 2 in its launcher constant [cf. (33)]. However, EMLs should not be compared in this manner, since their physical size may be quite different indicating a difference in volumetric efficiency. To factor in both electric-kinetic conversion efficiency and volumetric efficiency, EML comparisons should be done with equal bore diameter, bore length, armature mass, and armature velocity. A comparison under these conditions is termed a *same-scale* comparison.

IV. Experimental Results

This section presents new and recently published experimental results by the authors with conventional railgun, augmented railgun [4], and helical gun EML geometries [5]–[8]. The first experimental data set is from a one-turn augmented railgun (ARG). The ARG launcher has a 40 mm bore diameter, 750 mm bore length, 350 gram armature mass, and is powered by a single module pulse forming network (i.e., PFN) operating in ZC mode. Table I lists the PFN charge voltage, peak armature current, armature velocity, and measured electric-kinetic efficiency for each of the ARG experiments. Experimentally measured efficiency is given by

$$\eta = \frac{W_k}{W_u} = \frac{\frac{1}{2}mv_{\max}^2}{W_0 - W_f} \quad (35)$$

where W_k is the kinetic energy of the projectile, W_u is the total electrical energy used, W_0 is the initial electrical energy stored

TABLE I
AUGMENTED RAILGUN (ARG) EXPERIMENTAL RESULTS

Experiment	V_{charge} [v]	I_{peak} [kA]	v_{max} [m/s]	η
1.1	1700	204	86.4	0.061
1.2	1700	204	86.4	0.061
1.3	1700	204	88.4	0.063
2.1	1900	226	105.2	0.072
2.2	1900	226	104.9	0.071
2.3	1900	226	103.6	0.070
3.1	2100	255	125.8	0.084
3.2	2100	255	125.5	0.083
3.3	2100	255	124.7	0.082
4.1	2300	270	141.8	0.089
4.2	2300	270	141.2	0.088
4.3	2300	270	139.7	0.086

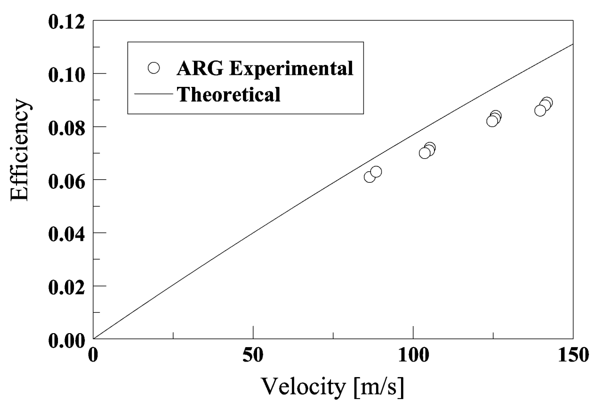

Fig. 4. Illustrating velocity-dependent efficiency for a one-turn augmented railgun.

in the PFN, and W_f is any electrical energy remaining in the PFN that is not used.

The first part of the analysis is an examination of efficiency versus velocity using the ARG data from Table I. The measured efficiency and theoretical efficiency of (19) are plotted in Fig. 4 versus velocity. The launcher constant used for plotting (19) is 300 [m/s] and is derived from static measurements of the inductance gradient ($L' = 1.2\ \mu$H/m) and average system resistance ($R = 0.4$ mΩ) although both of these parameters are known to vary during the experiment. As can be seen in Fig. 4, the velocity-dependent efficiency effect predicted by (19) is clearly evident. The ARG efficiency increases with velocity. The theoretical results are in good agreement with the experimental data at low velocity. There is 16.3% error between the predicted and measured results at the highest velocity. While this error is acceptable, it is attributed to increased system resistance from joule heating or decreased inductance gradient from high-frequency skin effects. Both of these effects are present at high velocity because of the high current and because of the so-called velocity skin-effect [9].

The second part of the experimental data analysis is a comparative analysis of *same-scale* EMLs. Table II is a performance summary of a helical gun, a one-turn augmented railgun, and an ideal conventional railgun. Although there is some variation in the armature mass, the EMLs are considered same-scale with nominal 40 mm bore diameter, 750 mm bore length, 500 gram armature (i.e., projectile) mass, and 150 m/s velocity. Table II lists launcher specifications and experimentally measured data as well as static measurements of the inductance gradient and average system resistance.

The LCG-6 and LCG-7 data of Table II are helical gun experiments conducted with mechanically identical armatures. The difference between the armatures is the LCG-7 armature is liquid nitrogen cooled to reduce its resistance, whereas the LCG-6 armature is room-temperature with no cooling. The liquid nitrogen cooling reduced the armature resistance from 8.0 mΩ to 1.3 mΩ, a factor of almost 8 [8]. The armature resistance decrease reduces the system resistance approximately 40% (the stator resistance constitutes approximately 50% of the system resistance). The σ and λ values are directly proportional to the system resistance and are similarly reduced.

The CRG data of Table II are from a simulation of an ideal conventional railgun. The ideal CRG simulation is frictionless, lossless, and powered with an ideal constant-current source. While constructing a launcher to meet these specifications would be difficult, the absence of same-scale railgun investigations in the literature dictated the need for the simulation. The CRG inductance gradient and system resistance are conservative estimates based on [10] and the authors' experience with the ARG.

Pulsed power supplies for the LCG and ARG EMLs are capacitor based pulse forming networks (PFNs). The interested reader should consult [11] for PFN construction details. The $\mathrm{V_{charge}}$ data of Table II is the PFN charge voltage. The LCG-6 and LCG-7 experiments use an eight-module PFN and, therefore, had eight different charging voltages. The maximum and minimum module charge voltages are given in Table II. The ARG experiment used a single-module PFN, as stated previously.

The Table II data show the LCG-6 and LCG-7 EMLs to have an inductance gradient more than 2 orders of magnitude greater than the ARG and CRG launchers. In addition, the σ and λ values for LCG-6 and LCG-7 are more than an order of magnitude lower than the ARG and CRG σ and λ values which means the LCG will be more efficient at fixed velocity, a fact verified in Table II. LCG-6 and LCG-7 are the most efficient launchers in Table II at 18.2% and 32%, respectively, and are the most efficient ever reported at this scale. The agreement between theoretical and experimental efficiency is good with a maximum error of 16.6% and a minimum error is 0% (exact agreement) with these errors attributed to changes in the σ and λ due to joule heating and/or skin effects. Thom and Norwood [12] also postulate that commutation effects could lower the effective inductance gradient of helical coil launchers.

Table II also lists the V–I operating characteristics of the various launchers. The LCG peak current is more than 20 times lower than the ARG peak current while accelerating a 40% larger mass. The maximum LCG PFN charge voltage is approximately three times lower than the ARG voltage. This, however, is misleading given the ARG operates in ZC mode. The ARG charge voltage would be comparable to the LCG voltage if it were operated in CC mode.

The CRG current is 16 times higher than the LCG current. The CRG operating voltage (operating voltage is used instead of PFN charge voltage since the CRG is driven with an ideal

TABLE II
ELECTROMAGNETIC LAUNCHER PERFORMANCE COMPARISON

Parameter	LCG-6	LCG-7	ARG	CRG
Bore diameter [mm]	40	40	40	40
Bore length [mm]	750	750	750	750
Projectile mass [g]	526	515	350	500
Inductance gradient [μH/m]	113	148	1.2	0.45
Operating mode	CC	CC	ZC	CC
R (min) [mΩ]	18.1	11.3	0.4	0.4
R (max) [mΩ]	21.9	12.1	2.0	0.4
R (avg) [mΩ]	20.0	11.7	0.4	0.4
λ [m/s] (Eq 32 or 33)	88	40	300	889
σ [m/s]	176	80	1200	1778
I_{peak} [kA]	12.4	11.5	270	183
V_{charge} [V]	300 to 550	250 to 550	2300	98
v_{max} [m/s]	137	164	141	150
Theoretical efficiency (Eq 19, 23, 29, 30) [%]	21.8	33.7	7.2	3.9
Measured efficiency [%]	18.2	32.0	8.8	3.9
Efficiency error [%]	16.6	5.1	16.3	0.0

LCG = long (i.e., helical) gun; ARG = augmented railgun; CRG = simulated ideal conventional railgun. See text for complete description of experiments.

current source) is a factor of 5.6 lower than the maximum LCG voltage. It is only a factor of 2.6 lower than the minimum LCG voltage. Caution is used when interpreting this result since the CRG is powered with an ideal current source. A system resistance increase of 1 mΩ would increase the operating voltage 183 V from ohmic voltage drop (since $I = 183$ kA). And considering that current is constant, joule heating could easily increase the resistance by this amount. Table II data show the CRG is the most inefficient launcher considered in this investigation. This is not surprising given its σ of almost 1800 m/s. The large current needed for this velocity would almost certainly cause significant joule heating leading to larger σ and λ and, ultimately, lower efficiency. The combined evidence suggests that low-σ and low-λ launchers can not only be operated at significantly lower currents but at voltage levels that are slightly higher than (given an ideal power source) or comparable with (given a nonideal power source) low-gradient launchers.

V. SUMMARY

This investigation develops a general theoretical efficiency and scaling relationship for constant gradient EMLs from basic principles expressing those relationships in terms of electrical circuit parameters. The efficiency and scaling relationships are similar for these types of launchers since force is produced by the gradient of self or mutual inductance. Expressions for electromagnetic force, back-voltage, and kinetic power are also developed and given in terms of electrical circuit parameters.

EML efficiency is shown to be a function of the armature velocity and the launcher's characteristic velocity. The characteristic velocity completely characterizes the launcher since it is the product of the mode constant and the launcher constant. The EML must operate at its characteristic velocity to achieve 50% maximum theoretical efficiency.

The launcher constant is the ratio of the system resistance and inductance gradient. The launcher constant is a scaling factor and a figure of merit which also characterizes the EML. As a scaling factor, it can be used to predict performance gains derived through changes in the EML geometry (i.e., system resistance and inductance gradient). As a figure of merit and with fixed operating mode and armature velocity, the launcher constant is useful when comparing launchers of different geometry.

The two modes of EML operation considered in this investigation are; constant current mode and zero exit current mode. The mode constant reflects the operating mode and determines the maximum EML efficiency. The maximum EML efficiency in constant current mode is 50% while the maximum EML efficiency in zero exit current mode is 100%. Inductive energy is stored in the launcher in constant current mode. Zero exit current mode allows any inductively stored energy to be used toward accelerating the armature or to be removed from the system.

The concept of an ideal launcher is developed in this investigation. The ideal launcher operates at 100% of its maximum theoretical efficiency at all velocities. A low-σ or low-λ geometry approximates the ideal launcher. This investigation also shows that EML comparisons should be done on a same-scale basis, meaning equal bore diameter, bore length, armature mass, and velocity. Same-scale comparisons account for both electric-kinetic conversion efficiency and volumetric efficiency.

A comparative analysis of a same-scale conventional railgun, augmented railgun, and helical gun is presented. The comparative analysis verifies that efficiency is a function of armature velocity and shows that low-σ or low-λ geometries, such as the helical gun, are many times more efficient than conventional and augmented railguns. Furthermore, the comparative analysis shows that low-σ or low-λ EMLs can operate at an order of magnitude lower current and with voltage comparable to or slightly higher than conventional and augmented railguns. High-efficiency EML geometries are desirable from a systems point of view since they reduce the primary power requirements, the size of the PFN, the switching requirements and, although not investigated here, the cooling requirements and lifetime of the launcher.

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